

High-mass twin stars with a multi-polytrope EoS

D. E. Alvarez-Castillo^{1,2,*} and D. B. Blaschke^{1,3,4,†}

¹ *Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research,
Joliot-Curie str. 6, 141980 Dubna, Russia*

² *Instituto de Física, Universidad Autónoma de San Luis Potosí
Av. Manuel Nava 6, San Luis Potosí, S.L.P. 78290, México*

³ *Institute for Theoretical Physics, University of Wrocław, Max Born Pl. 9, 50-204 Wrocław, Poland*

⁴ *National Research Nuclear University (MEPhI), Kashirskoe Shosse 31, 115409 Moscow, Russia
(Dated: March 9, 2017)*

We show that in the 3-polytropes model of Hebeler et al. [1] for the neutron star equation of state at supersaturation densities a third family of compact stars can be obtained which confirms the possibility of high-mass twin stars that have coincident masses $M_1 = M_2 \approx 2 M_\odot$ and significantly different radii $|R_1 - R_2| > 2 - 3$ km. We consider a scenario of a first order phase transition which eliminates one of the three polytropes from the star structure and results in a sharp boundary between a high-density and low-density phase.

PACS numbers: 04.40.Dg, 12.38.Mh, 26.60.+c, 97.60.Jd

I. INTRODUCTION

Neutron stars (NS) are extreme compact objects that represent one possible end point of stellar evolution. The very high density conditions in their interiors are described by an equation of state (EoS) which is currently undetermined, posing several questions on the NS composition. In particular, the existence of quark matter in their cores could answer some questions about the topology of the QCD phase diagram, namely regarding phase transitions in the high density, low temperature regime. In case of a first order phase transition in NS, the high-mass twin (HMT) phenomenon [2, 3] predicts an EoS featuring astrophysical observables that are reflected in the corresponding mass-radius (M-R) diagram. The so-called *third family* branch corresponds to hybrid neutron stars composed of a hadronic mantle and quark matter core, whereas the second family branch of purely hadronic stars is separated from it by a sequence of unstable configurations. Thus, within the HMT phenomenon, two stars of the same mass would be located in the second and third family branches respectively, having different radii and internal composition. It is expected that upcoming observational astronomy missions, like NICER [4] or SKA [5], may be able to search for the HMT phenomenon, having the sufficient accuracy to resolve the different NS radii of the twin stars. As pointed out in [6], the HMT phenomenon is of great relevance for the study of the NS EoS not only because it can provide evidence for a first order phase transition and thus for the very existence of a critical endpoint in the QCD diagram, but also because it provides a resolution to several issues: the hyperon puzzle, the reconfinement problem and the masquerade case (see [6] and references therein). In addition,

the HMT may be discussed in the context of explaining the origin of fast radio bursts [7] as possible intermediate metastable states due to a sudden change in the internal structure of a fast rotating supramassive neutron star [8, 9] created, e.g., in a NS merger event before its final collapse to a black hole [10].

The purpose of this rapid communication is to point out that the HMT case is not the result of the construction of a rather exotic case of an EoS but may be obtained even within the rather conservative scheme of Hebeler et al. [1] that consists of a multi-polytrope description of the NS EoS [11] in line with constraints derived from a chiral effective field theory describing nuclear few- and many-particle systems at densities up to nuclear saturation.

II. PIECEWISE POLYTROPE EOS WITH A FIRST-ORDER PHASE TRANSITION

We would like to investigate the question whether in the scheme of Hebeler et al. [1] with piecewise polytrope EoS at supersaturation densities it would be possible to describe the HMT phenomenon. To this end one should define one of the polytropes as a constant pressure region with $P = P_{\text{crit}}$ with a jump in energy density $\Delta\varepsilon$ due to a first order phase transition that would fulfil the Seidov constraint [12]

$$\frac{\Delta\varepsilon}{\varepsilon_{\text{crit}}} \geq \frac{1}{2} + \frac{3}{2} \frac{P_{\text{crit}}}{\varepsilon_{\text{crit}}} \quad (1)$$

for the occurrence of an instability in the M-R relation of compact stars. Such a sequence of unstable configurations is precondition for a disconnected (third family) branch of stable hybrid stars that would furthermore require a sufficiently stiff high density EoS to allow for a maximum mass fulfilling the constraint from the measurement of the mass $M = 2.01 \pm 0.04 M_\odot$ for the pulsar

*Electronic address: alvarez@theor.jinr.ru

†Electronic address: blaschke@ift.uni.wroc.pl

PSR J0348+0432 [13]. According to [1], the supersaturation density region is split into three regions

$$\begin{aligned} i = 1 & : n_1 \leq n \leq n_{12} \\ i = 2 & : n_{12} \leq n \leq n_{23} \\ i = 3 & : n \geq n_{23}, \end{aligned} \quad (2)$$

where $n_1 = 1.1 n_0$ is just above the nuclear saturation density $n_0 = 0.15 \text{ fm}^{-3}$ and the polytrope EoS pieces fulfill

$$P_i(n) = \kappa_i n^{\Gamma_i}$$

in the corresponding regions. For our setting of the problem, in all our EoS models the first region is taken from the Hebeler et al. paper [1] and corresponds to a polytrope fit to the stiffest EoS ($n > 1.1 n_0$) of their table V together with an intermediate homogeneous phase in β -equilibrium ($0.5 n_0 < n < 1.1 n_0$), presented in their section III, and the BPS EoS for the outer NS crust ($n < 0.5 n_0$) of their table VII. Therefore, the resulting fit gives polytrope parameter values for this density region of $\Gamma_1 = 4.92$ and $\kappa_1 = 17906.60 \text{ MeV} \cdot \text{fm}^{3(\Gamma_1-1)}$. Furthermore, the region $i = 2$ shall correspond to the phase coexistence region of the first order phase transition with constant pressure, so that $\Gamma_2 = 0$ and $P_2 = \kappa_2 = P_{\text{crit}}$. The boundaries of this region shall be obtained from a Maxwell construction which requires the pressure as a function of the chemical potential μ . To facilitate this construction for a pair of polytropes at zero temperature, we utilize the formulas given in the Appendix of Ref. [14],

$$P(n) = n^2 \frac{d(\varepsilon(n)/n)}{dn}, \quad (3)$$

$$\begin{aligned} \varepsilon(n)/n &= \int dn \frac{P(n)}{n^2} = \int dn \kappa n^{\Gamma-2} \\ &= \frac{\kappa n^{\Gamma-1}}{\Gamma-1} + C, \end{aligned} \quad (4)$$

$$\mu(n) = \frac{P(n) + \varepsilon(n)}{n} = \frac{\kappa \Gamma}{\Gamma-1} n^{\Gamma-1} + m_0, \quad (5)$$

where the integration constant C is fixed by the condition that $\varepsilon(n \rightarrow 0) = m_0 n$. Now we have for the polytrope EoS

$$n(\mu) = \left[(\mu - m_0) \frac{\Gamma-1}{\kappa \Gamma} \right]^{1/(\Gamma-1)}, \quad (6)$$

$$P(\mu) = \kappa \left[(\mu - m_0) \frac{\Gamma-1}{\kappa \Gamma} \right]^{\Gamma/(\Gamma-1)}. \quad (7)$$

III. HIGH MASS TWINS FROM MULTI-POLYTROPE EQUATIONS OF STATE

Now we can apply these general relations to the case of a transition from nuclear matter in the region 1 (with $m_{0,1}$ being the nucleon mass) to high density matter in

region 3. That may correspond, e.g., to hyperon matter or quark matter. From the Maxwell construction

$$P_1(\mu_{\text{crit}}) = P_3(\mu_{\text{crit}}) = P_{\text{crit}} \quad (8)$$

$$\mu_{\text{crit}} = \mu_1(n_{12}) = \mu_3(n_{23}) \quad (9)$$

follow the two conditions

$$\kappa_3 = \kappa_1 n_{12}^{\Gamma_1} / n_{23}^{\Gamma_3}, \quad (10)$$

$$\begin{aligned} P_{\text{crit}} &= (m_{0,1} - m_{0,3}) \left[\frac{\Gamma_1}{n_{12}(\Gamma_1 - 1)} - \frac{\Gamma_3}{n_{23}(\Gamma_3 - 1)} \right]^{-1} \\ &= \kappa_1 n_{12}^{\Gamma_1}. \end{aligned} \quad (11)$$

The above equations (10) and (11), allow for determination of κ_3 and $m_{0,3}$ once the values of n_{12} , n_{23} , $m_{0,1}$ and Γ_3 are fixed. In order to fulfill the compact star mass constraint, we may demand that upon solving the corresponding compact star sequence, the mass at the onset of the transition fulfills $M(n_{12}) \simeq 2 M_{\odot}$ which fixes the value n_{12} , allowing to determine P_{crit} and μ_{crit} from the Maxwell construction. Moreover, since the EoS just above nuclear saturation is fixed according to the stiff limit of Hebeler et al. [1], the constants Γ_1 and κ_1 are also fixed. So we are then left with the three equations (10) and (11) and (1) for the four unknowns κ_3 , Γ_3 and n_{23} and $m_{0,3}$. We fix n_{23} such that the equality sign holds in the Seidov criterion (1). Therefore, in order to close the system, we dial Γ_3 as a free parameter. Its maximal value is determined so that the speed of sound shall not exceed the speed of light up to the density values reached in the very center of the maximum mass star configurations.

With the above scheme, we are thus able to compute the EoS of hybrid compact stars. Next, we obtain the corresponding star sequences by solving the Tolman-Volkoff-Oppenheimer (TOV) equations describing a static, non-rotating, spherically symmetric star [15, 16]

$$\frac{dP(r)}{dr} = - \frac{G(\varepsilon(r) + P(r)) (M(r) + 4\pi r^3 P(r))}{r(r - 2GM(r))} \quad (12)$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r), \quad (13)$$

with $P(r = R) = 0$ and $P_c = P(r = 0)$ as boundary conditions for a star with mass M and radius R . The complete NS sequence is determined by increasing the chosen central pressure P_c up to a maximum mass. The enclosed baryonic mass is obtained by integrating

$$\frac{dN_B(r)}{dr} = 4\pi r^2 \left(1 - \frac{2GM(r)}{r} \right)^{-1/2} n(r). \quad (14)$$

It plays an important role in the description of the dynamics of NS evolution scenarios.

In our numerical calculations, we choose three values of the polytrope index Γ_3 in order to discuss the effect of the stiffness of the high-density EoS. The corresponding sets 1-3 of model parameters share the same hadronic branch,

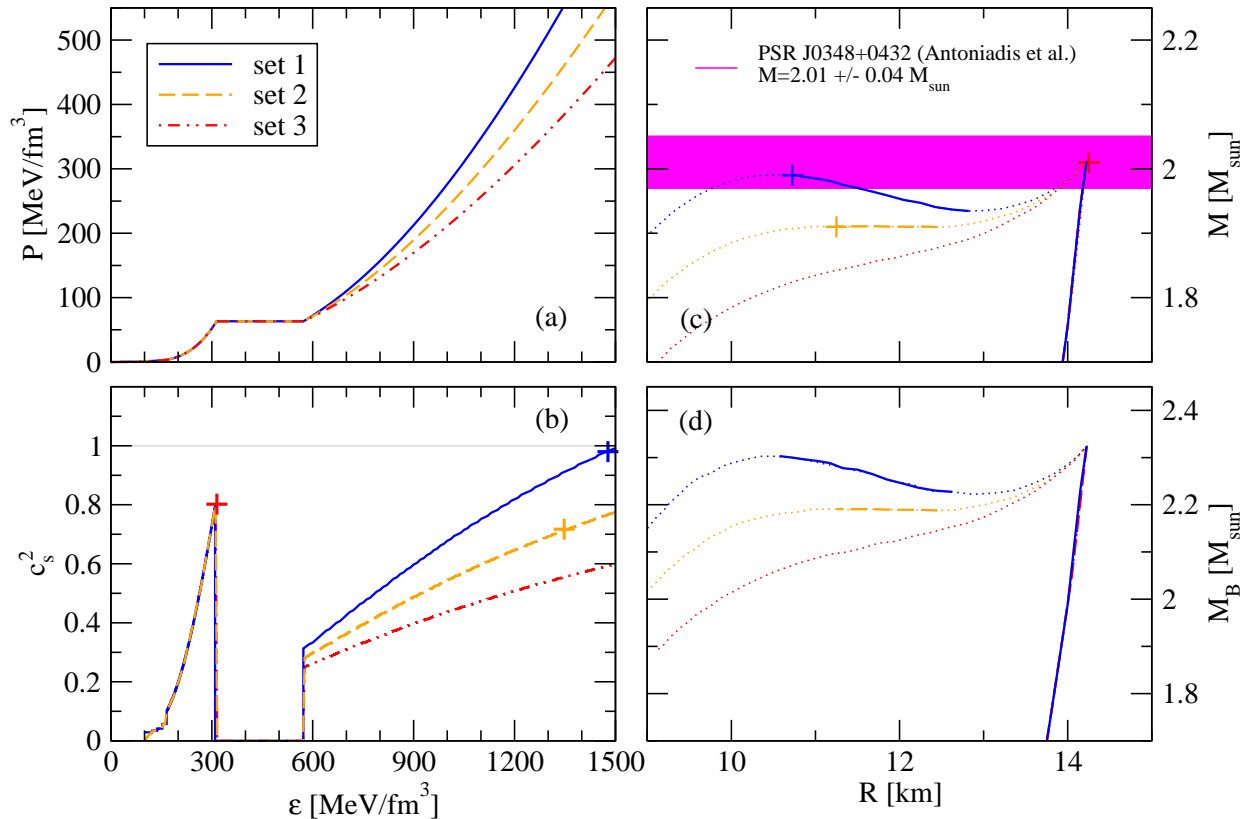


FIG. 1: (Color online) EoS ((a) - P vs. ε ; (b) - c_s^2 vs. ε) and sequences of compact stars ((c) - M vs. R ; (d) - M_B vs. R) for sets 1-3 of table I. The EoS have the same onset density and density jump of the phase transition, but different stiffness of the high density (quark matter) phase. The plus symbols denote values for the maximum mass configurations.

onset and density jump at the phase transition, see the panel (a) of Fig. 1. On panel (b) of Fig. 1 we demonstrate that the causality constraint is fulfilled for all three sets. The stiffest EoS, set 3, reaches the causality limit just at the maximum mass of the hybrid star sequence, which is denoted by the plus symbols on panels (b) and (c) of that figure. The set 3 is the main result of this paper as it shows that a third family of stable hybrid stars can be obtained within the multi-polytrope scheme of hebel et al. [1]. This branch has a maximum mass which even reaches $\sim 2 M_\odot$ and is more compact than the purely hadronic one by about 2 – 3 km. This is a potentially observable effect!

Lowering Γ_3 we obtain a value for which barely a stable hybrid star sequence can be obtained (set 2), and lowering Γ_3 further (set 1) no stable hybrid stars are possible. The three parameter sets for which the EoS and compact star sequences are illustrated in Fig. 1 are given in Tab. I.

TABLE I: Parameter values for sets 1, 2 and 3. The EoS in this set share the following properties: $P_{\text{crit}} = 63.18 \text{ MeV fm}^{-3}$, $\varepsilon_{\text{crit}} = 318.26 \text{ MeV fm}^{-3}$, $\Delta\varepsilon = 253.89 \text{ MeV fm}^{-3}$. The second polytrope with $P_2 = P_{\text{crit}}$ and $\Gamma_3 = 0$ lies between the densities $n_{12} = 0.32 \text{ fm}^{-3}$ and $n_{23} = 0.53 \text{ fm}^{-3}$.

	Γ_3	κ_3 [MeV fm $^{3(\Gamma_3-1)}$]	$m_{0,3}$ [MeV]	M_{max}^{NS} [M_\odot]	M_{max}^{HS} [M_\odot]	M_{min}^{HS} [M_\odot]
set 1	3.12	447.16	1014.87	2.01	1.991	1.934
set 2	2.80	365.12	1004.88	2.01	1.910	1.909
set 3	2.50	302.56	991.75	2.01	-	-

All the EoS models in this work are causal and fall inside the EoS region that was given in Hebeler et al. [1] for the case supporting a $1.97 M_\odot$ NS, see Fig. 2.

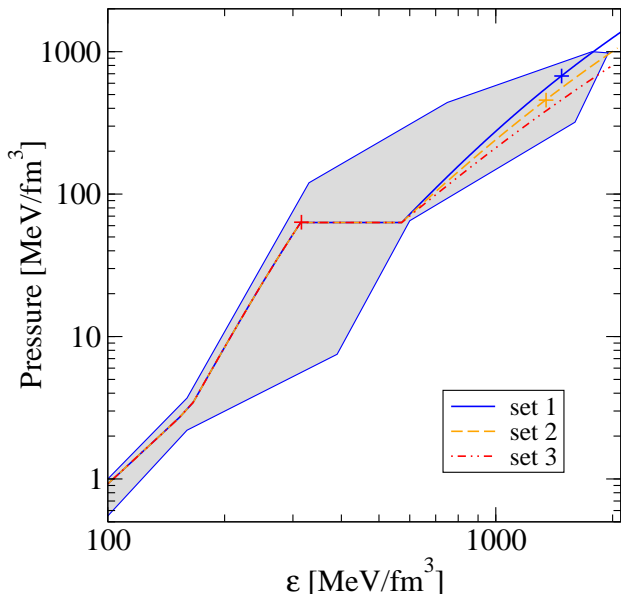


FIG. 2: (Color online) All multi-polytrope EoS for sets 1-3 from Tab. I fall within the region given in Hebeler et al. [1] for the case supporting a $1.97 M_{\odot}$ NS (grey shaded region). These EoS share the hadronic branch EoS, the onset density and jump in energy density at the transition but vary in the stiffness of the high-density phase. The plus symbols denote the pressure and energy density values at the center of the compact star configuration with the maximum mass.

IV. CONCLUSIONS

In this rapid communication we have shown that within the multi-polytrope approach by Hebeler et al. [1] we can obtain high mass twins in the M-R diagram for compact stars. The EoS parametrizations presented here obey causality and fall in the constraint region derived in [1] for the condition that a mass of $1.97 M_{\odot}$ has to be reached. In concluding, we mention a limitation of the multi-polytrope approach which cannot replace a realistic EoS. Following the three-polytrope scheme the hybrid star configurations could not be reached within evolutionary scenarios of mass accretion from a companion star or by spin down starting from the hadronic branch. This is because no hybrid EoS parametrization could be found for which the maximum baryonic mass on the hybrid star branch is higher than that of the most massive purely hadronic star. The main reason being the polytrope index of the high-density region cannot exceed a limiting value dictated by the causality constraint on the EoS. Microscopic approaches to high-density quark matter like, e.g., the NJL model with higher order repulsive interactions [2] or the relativistic string-flip model [17] do not have this problem of the multi-polytrope approach.

Acknowledgements

We acknowledge discussions with Jim Lattimer and Jochen Wambach. D.B. is grateful for support of his participation at the CERN Theory Institute "From quarks to gravitational waves", where these results have first been presented. This work was supported by the Polish NCN under grant No. UMO-2014/13/B/ST9/02621 (Opus7).

-
- [1] K. Hebeler, J. M. Lattimer, C. J. Pethick and A. Schwenk, *Astrophys. J.* **773**, 11 (2013).
 - [2] S. Benic, D. Blaschke, D. E. Alvarez-Castillo, T. Fischer and S. Typel, *Astron. Astrophys.* **577**, A40 (2015).
 - [3] D. Alvarez-Castillo, S. Benic, D. Blaschke, S. Han and S. Typel, *Eur. Phys. J. A* **52**, no. 8, 232 (2016).
 - [4] <https://heasarc.gsfc.nasa.gov/docs/nicer>
 - [5] <http://www.ska.ac.za>
 - [6] D. Blaschke and D. E. Alvarez-Castillo, *AIP Conf. Proc.* **1701**, 020013 (2016).
 - [7] D. J. Champion *et al.*, *Mon. Not. Roy. Astron. Soc.* **460**, no. 1, L30 (2016)
 - [8] D. E. Alvarez-Castillo, M. Bejger, D. Blaschke, P. Haensel and L. Zdunik, [arXiv:1506.08645](https://arxiv.org/abs/1506.08645) [astro-ph.HE].
 - [9] M. Bejger, D. Blaschke, P. Haensel, J. L. Zdunik and M. Fortin, *Astron. Astrophys.*, to appear (2017); [arXiv:1608.07049](https://arxiv.org/abs/1608.07049) [astro-ph.HE].
 - [10] H. Falcke and L. Rezzolla, *Astron. Astrophys.* **562**, A137 (2014).
 - [11] J. S. Read, B. D. Lackey, B. J. Owen and J. L. Friedman, *Phys. Rev. D* **79**, 124032 (2009)
 - [12] Z. F. Seidov, *Sov. Astron.* **15**, 347 (1971).
 - [13] J. Antoniadis *et al.*, *Science* **340**, 6131 (2013).
 - [14] J. L. Zdunik, M. Bejger, P. Haensel and E. Gourgoulhon, *Astron. Astrophys.* **450**, 747 (2006).
 - [15] R. C. Tolman, *Phys. Rev.* **55**, 364 (1939).
 - [16] J. R. Oppenheimer and G. M. Volkoff, *Phys. Rev.* **55**, 374 (1939).
 - [17] M. A. R. Kaltenborn, N. U. F. Bastian and D. B. Blaschke, [arXiv:1701.04400](https://arxiv.org/abs/1701.04400) [astro-ph.HE].